Translation and Rotation

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Outline

1. You have two choices
2. Applying a formula to each point
   - Example of translating a point
   - Example of rotating a point
   - Other notes about rotating and translating points
3. Homogeneous coordinates: using matrix multiplication
   - Format
   - Rotation
   - Translation
   - Putting it all Together

You have two choices

- For Assignment 2, you will have to write your own code to rotate and translate figures.
- You cannot use move.h or libmove.a.
- You have two choices. You can either
  - apply a formula to each point in a figure, or
  - treat figures as matrices, and use matrix multiplication.

Applying a formula to each point

- We will show you how to rotate or translate one point.
- You can then apply this to all points in your C code for Assignment 2.
- To translate the point (x,y) by delta_x and delta_y:
  $translated\_x = x + delta\_x$
  $translated\_y = y + delta\_y$
- To rotate the point (x,y) by *angle:*
  $rotated\_x = x \times cos(angle) - y \times sin(angle)$
  $rotated\_y = x \times sin(angle) + y \times cos(angle)$

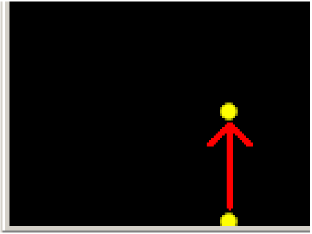
Example of translating a point

To translate the point (100, 2) by 50 vertically and zero horizontally, figure out the new x value

$$translated\_x = x + delta\_x$$
$$translated\_x = 100 + 0 = 100$$

and the new y value

$$translated\_y = y + delta\_y$$
$$translated\_y = 2 + 50 = 52$$



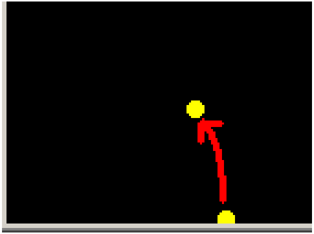
Example of rotating a point

To rotate the point (100, 2) by $\frac{PI}{6}$ radians (30°) counter-clockwise about the origin (0,0), figure out the new x value

$$rotated\_x = x \times cos(angle) - y \times sin(angle)$$
$$rotated\_x = 100 \times cos(\frac{PI}{6}) - 2 \times sin(\frac{PI}{6})$$

and the new y value

$$rotated\_y = x \times sin(angle) + y \times cos(angle)$$
$$rotated\_y = 100 \times sin(\frac{PI}{6}) + 2 \times cos(\frac{PI}{6})$$



Homogeneous coordinates: using matrix multiplication

Motivation

- Not necessary for 201, but it helps with Asn 2, and is an introduction to graphics.
- In computer graphics, images are defined as a set of points (among other things)
- Often, we want to animate these objects using:
  - Rotations (Spin)
  - Translations (Move)
- Homogeneous coordinates allow us to easily combine these animation tasks
  - Everything can be handled with matrix multiplication

Homogeneous Coordinates at a Glance

- Commonly, a 2D point is represented in the form $p = \begin{bmatrix} x & y \end{bmatrix}$
- To make matrix multiplication possible, we represent a 2D point as $p = \begin{bmatrix} x & y & 1 \end{bmatrix}$
- We can then store our points in a matrix of the following form and use matrix multiplication to move objects

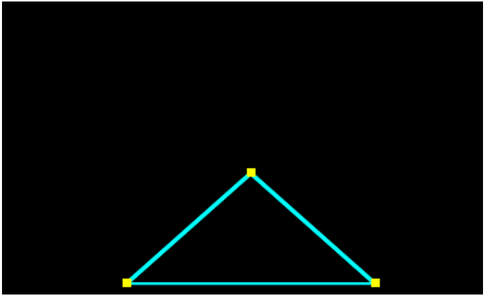
$$M = \begin{bmatrix} x1 & y1 & 1 \\ x2 & y2 & 1 \\ . & . & . \\ . & . & . \\ xn & yn & 1 \end{bmatrix}$$

- Note you should already be storing points in a similar format.

Homogeneous Coordinates Examples

For our examples we will use the following triangle

$$M = \begin{bmatrix} 100 & 10 & 1 \\ 200 & 100 & 1 \\ 300 & 10 & 1 \\ 100 & 10 & 1 \end{bmatrix}$$



Rotation using Homogeneous Coordinates

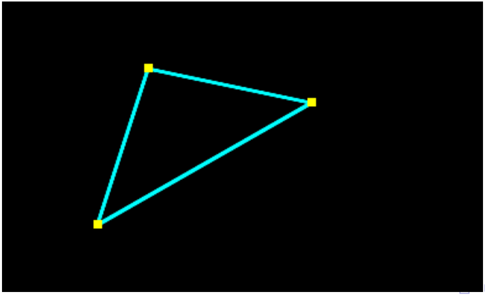
- We apply a rotation to an object with the following matrix multiplication
  - θ is the angle of rotation (around the origin)
  - M(rot) will always have size $n \times 3$

$$M(rot) = \begin{bmatrix} x1 & y1 & 1 \\ x2 & y2 & 1 \\ x3 & y3 & 1 \\ . & . & . \\ . & . & . \\ . & . & . \\ xn & yn & 1 \end{bmatrix} \times \begin{bmatrix} cos(\theta) & sin(\theta) & 0 \\ -sin(\theta) & cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotation Example

Let's rotate our triangle by $\frac{PI}{6}$ radians (30°)

$$M(rot) = \begin{bmatrix} 100 & 10 & 1 \\ 200 & 100 & 1 \\ 300 & 10 & 1 \\ 100 & 10 & 1 \end{bmatrix} \times \begin{bmatrix} cos(\frac{PI}{6}) & sin(\frac{PI}{6}) & 0 \\ -sin(\frac{PI}{6}) & cos(\frac{PI}{6}) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Translation using Homogeneous Coordinates

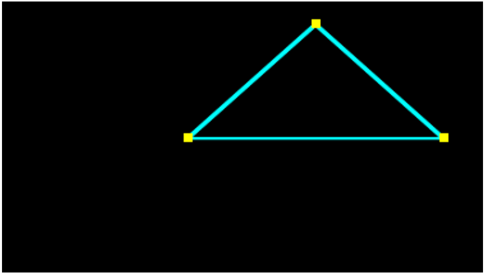
- We can translate an object with the following matrix multiplication
  - Tx and Ty are the translation distances in x and y

$$M(tran) = \begin{bmatrix} x1 & y1 & 1 \\ x2 & y2 & 1 \\ x3 & y3 & 1 \\ . & . & . \\ . & . & . \\ . & . & . \\ xn & yn & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ Tx & Ty & 1 \end{bmatrix}$$

Translation Example

Let's move our triangle 50 pixels right and 100 pixels up

$$M(tran) = \begin{bmatrix} 100 & 10 & 1 \\ 200 & 100 & 1 \\ 300 & 10 & 1 \\ 100 & 10 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 50 & 100 & 1 \end{bmatrix}$$



The Power of Homogeneous Coordinates

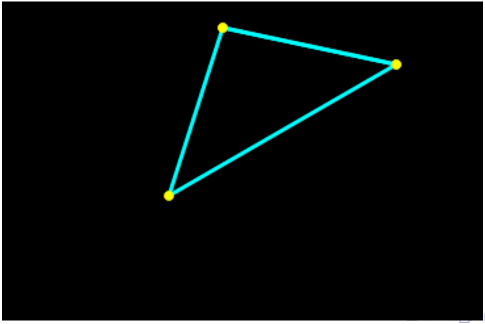
- The great thing about homogeneous coordinates is that we can easily combine rotations and translations to do more complex movement
- For example, let's rotate our triangle $\frac{PI}{6}$ radians, and move it 50 pixels to the right, 100 pixels up.

$$M = \begin{bmatrix} 100 & 10 & 1 \\ 200 & 100 & 1 \\ 300 & 10 & 1 \\ 100 & 10 & 1 \end{bmatrix} \begin{bmatrix} cos(\frac{PI}{6}) & sin(\frac{PI}{6}) & 0 \\ -sin(\frac{PI}{6}) & cos(\frac{PI}{6}) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 50 & 100 & 1 \end{bmatrix}$$

Example Cont.

We multiply the rotation and translation matricies and get

$$M = \begin{bmatrix} 100 & 10 & 1 \\ 200 & 100 & 1 \\ 300 & 10 & 1 \\ 100 & 10 & 1 \end{bmatrix} \times \begin{bmatrix} cos(\frac{PI}{6}) & sin(\frac{PI}{6}) & 0 \\ -sin(\frac{PI}{6}) & cos(\frac{PI}{6}) & 0 \\ 50 & 100 & 1 \end{bmatrix}$$



Summary and other notes

- You can use either method: translate and rotate individual points, or use homogenous coordinates and matrix multiplication.
- Remember to convert degrees to radians, and to use the PI macro.
- For more information, most of which is not needed for 201, see the Hints section at the bottom of Assignment 2.